University of Sargodha

BS 4th Term Examination 2024

Subject: Computer Science **Paper: Linear Algebra (MATH-202/MATH-3215)**

Time Allowed: 02:30 Hours **(For Regular & Retake Students)** **Maximum Marks: 60**

Note: **Objective part is compulsory. Attempt any three questions from subjective part.**

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines of each on your answer sheet. (2*12)

i. Define trace of a matrix.

ii. Whether the vectors $u_1 = (1,2,-3), u_2 = (1,-4,3)$, are orthogonal or not.

iii. Write the bases for the vector space $M_{2\times2}$ of 2×2 matrices.

iv. Let V be vector space and $u \in V$ then show that $(-1)u = -u$.

v. If A is invertible matrix, then A^T is also invertible and $(A^T)^{-1} = (A^{-1})^T$.

vi. Define Markove Matrix.

vii. Find inverse of $A = \begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}$.

viii. Let V be a vector space over a field K. Show that for any scaler k and $0 \in V, K0 = 0$.

ix. Show that set of all matrices with trace zero is subspace of vector space of all $n \times n$ matrices.

x. If $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, show that $A^4 = I_2$.

xi. Write the basis of $P_n(x)$.

xii. Define similar matrices.

Subjective Part (3*12)

Q.2. (a) Determine whether $(1,1,1,1), (1,2,3,2), (2,5,6,4), (2,6,8,5)$ form basis of R^4. If not, find the dimension of the subspace they span.

(b) Show that matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ satisfy its characteristic equation.

Q.3. (a) Let W be subspace of R^5 spanned by the vectors $u_1 = (1,2,-1,3), u_2 = (2,4,1,-2)$, $u_3 = (3,6,3,-7),\ u_4 = (1,2,-4,11), u_5 = (2,4,-5,14)$. Find basis and dimension of W.

(b) Find Eigen values and corresponding Eigen vectors of $A = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$.

Q.4. (a) Consider the vectors $u_1 = (1,2,1,3,2),\ u_2 = (1,3,3,5,3),\ u_3 = (3,8,7,13,8)$, $w_1 = (1,4,6,9,7),\ w_2 = (5,13,13,25,19)$ in R^4, let $U = span(u_i), w = span(w_i)$. Then show that $U = W$

(b) Solve the following system of Linear equations by using Row Operation

$$x + y + 2z = 9$$
$$2x + 4y - 3z = 1$$
$$3x + 6y - 5z = 0$$

Q.5. (a) Find A^{-1}, if $A = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{bmatrix}$.

(b) Apply the Gram-Schmidt process to find an orthogonal basis and then an orthonormal basis for t the subspace U of R^4 spanned by $u_1 = (1,1,1,1), u_2 = (1,2,4,5), u_3 = (1,-3,-4,-2)$.

Q.6. (a) If $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$ then diagonalize that matrix.

(b) Let $v_1 = (1,2,1),\ v_2 = (2,9,0)$ and $v_3 = (3,3,4)$. Show that the set $S = \{v_1, v_2, v_3\}$ is basis for $\mathbb{R}^3$.